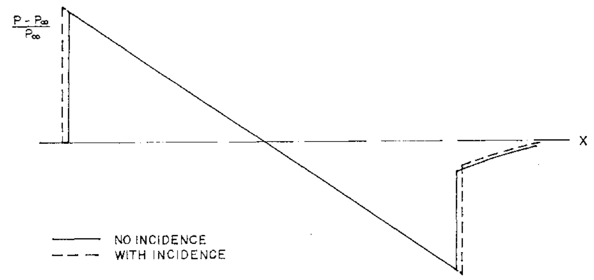

Fig. 2 Typical pressure signature.

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Propagation of Self-Similar Radiation Waves

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Introduction

IN a nuclear explosion, a significant fraction of the total energy of the bomb is released in the form of thermal radiation. There is an initial period of "radiation expansion," as called by Magee and Hirschfelder,[1] that is characterized by having the air heated to a high temperature by radiation before it is set into motion by hydrodynamical signals from the explosion. Considering radiation as the predominant mechanism for energy transfer during an explosion, Lall and Viskanta[2] recently solved the problem of the expansion of a spherical mass of compressible gas by using a method of finite differences. Their numerical result shows that initially the gas density remains almost constant with respect to time.

Employing constant density approximation, Andriankin[3] studied the propagation of a spherical thermal wave from an instantaneous point source in a gas at such high temperatures that the heat flux due to conduction was negligible compared with that due to radiation. The gas energy was expressed as the sum of radiation energy and material internal energy, but the complete solution for such a general case was not obtained because of the non-self-similar nature of the problem.

By assuming that the radiation energy was negligible compared with the internal energy, a similarity solution was obtained by Marshak[4] in studying a one-dimensional radiation wave penetrating into a cold medium of constant density. However, since the temperature at the wave front was prescribed by an exponential time-dependent function, and due to the nonsymmetric form of the solution in space coordinate, his problem is different from the one concerning the early stage of an explosion.

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Under the assumption of invariant density immediately after an explosion, it can be shown that self-similar solutions are possible if either the radiation energy is negligible compared with the material internal energy or vice versa. In the present analysis we assume that initially an amount of heat Q_0 is distributed in a high temperature region that is separated from the cold surrounding by an imaginary surface at a distance r_0 from the center of the coordinate system. The hot region has a symmetric plateau-like temperature distribution with initial central temperature T_0, whereas the temperature of the surrounding gas is so low compared with that of the hot gas that it can be considered as zero.

Fundamental Equations

For sufficiently high temperatures at which the heat transfer by conduction is negligible compared with that by radiation, and without any energy production within the gas, the equations of conservation of mass, momentum, and energy are[4]

$$(D\rho/Dt) + \rho \operatorname{div}\mathbf{u} = 0 \tag{1}$$

$$(D\mathbf{u}/Dt) + (1/\rho) \operatorname{grad}p = 0 \tag{2}$$

$$(DE_{\text{tot}}/Dt) + (1/\rho) \operatorname{div}(p\mathbf{u} + \mathbf{F}_R) = 0 \tag{3}$$

where ρ is the density; $\mathbf{u}$ is the fluid velocity; $E_{\text{tot}} = \frac{1}{2}u^2 + E_M + E_R/\rho$ is the total energy content per unit mass with E_M the material internal energy per unit mass, and E_R the radiation energy per unit volume; $p = p_M + p_R$ is the total pressure with p_M the material pressure, and p_R the radiation pressure; $\mathbf{F}_R = -D_R \operatorname{grad}E_R$ is the radiation flux with D_R the diffusion coefficient for radiation; and D/Dt is the substantial time derivative.

We consider only problems with a configuration having plane, cylindrical, or spherical symmetry. With the simplification of constant density, Eqs. (1) and (2) indicate that the gas is motionless and the total pressure is constant. Thus Eq. (3) reduces to

$$(\partial/\partial t)[E_M + (E_R/\rho)] = 1/\rho \operatorname{div}(D_R \operatorname{grad}E_R) \tag{4}$$

Following Marshak's paper, let us use the appropriate expressions for E_M, E_R, and D_R, namely, $E_M = CvT$ with Cv the specific heat at constant volume, and T the temperature; $E_R = \sigma T^4$ with σ the Stefan-Boltzmann constant; $D_R = lc/3$ with c the speed of light, and l the mean free path for radiation which is defined by $l = 1/(K\rho)$. K is the Roseland's mean opacity that usually can be written in the following form:

$$K = (1/l_0\rho_0)(\rho/\rho_0)^m(T_0/T)^n \tag{5}$$

where l_0 is the mean free path at density ρ_0 and temperature T_0, and m and n are positive numbers. Inserting the foregoing expressions into Eq. (4) we obtain

$$\frac{\partial}{\partial t}\left(CvT + \frac{\sigma T^4}{\rho}\right) = \frac{4}{3(n+4)}\frac{l_0 c\sigma}{\rho T_0^n} \times \left(\frac{\partial^2}{\partial r^2} + \frac{\nu - 1}{r}\frac{\partial}{\partial r}\right) T^{n+4} \tag{6}$$

where $\nu = 1,2,3$, respectively for planar, cylindrical, and spherical configuration.

Similarity Solution for $E_M \gg E_R$

Equation (6), appropriate to this case, has the following dimensionless form:

$$\partial T^*/\partial t^* = k[(\partial^2/\partial r^{*2}) + (\nu - 1)/r^*(\partial/\partial r^*)]T^{*n+4} \tag{7}$$

where $t^* = t/[(E_{M_0}/E_{R_0})(r_0^2/l_0c)]$ with $E_{M_0} = CvT$ and $E_{R_0} = \sigma T_0^4/\rho$, $r^* = r/r_0$, $T^* = T/T_0$ and $k = \frac{4}{3}/(n+4)$.

For the solution of Eq. (7), let us assume a general functional form

$$T^*(r^*,t^*) = A(t^*)f(z) \qquad z = B(r^*)C(t^*) \tag{8}$$

which reduces the equation to

$$\frac{A}{A'}\frac{C'}{C} zf' + f = k\frac{A^{n+4}}{A'}\left[B'^2C^2\frac{d^2f^{n+4}}{dz^2} + \left(B'' + \frac{\nu-1}{r^*}B'\right)C\frac{df^{n+4}}{dz}\right] \tag{9}$$

where a prime denotes differentiation with respect to the argument. Self-similar solutions are possible only when Eq. (9) does not contain r^* or t^* explicitly and can be reduced to an ordinary differential equation. To achieve this the following are assumed:

1) let $(A/A')(C'/C) = \alpha$, which gives

$$C = A^{\alpha} \tag{10}$$

2) let $B'' = 0$ and $B' = 1$, which gives

$$B = r^* \tag{11}$$

3) let $A^{n+4}C^2/A' = -1/\beta$. By using Eq. (10), it becomes $A'/A^{n+2\alpha+4} = -\beta$. If $n + 2\alpha + 4 = 0$, the function A varies linearly with t^*, and if $n + 2\alpha + 4 = 1$, it becomes an exponential function, which was the form used by Marshak.[4] Both cases can be shown to be unsuitable to the present analysis. For $n + 2\alpha + 4 \neq 0$ nor 1, we have

$$A = [1 + (n + 2\alpha + 3)\beta t^*]^{-1/(n+2\alpha+3)} \tag{12}$$

The value of α can be determined from the fact that the energy contained within the wave front should be a constant. In the present approximation with radiation energy ignored, we get

$$Q_0 = \begin{cases} \int_0^{r_f} CvT(r,t)r^{\nu-1}dr & \text{for } \nu = 1 \\ \int_0^{r_f} CvT(r,t)\pi(2r)^{\nu-1}dr & \text{for } \nu = 2,3 \end{cases} \tag{13}$$

where $r_f(t)$ is the location of the wave front. Expressed in terms of dimensionless variables and by using Eqs. (8, and 10–12), the right side of (13) can easily be shown to be a constant quantity only when

$$\alpha = 1/\nu \tag{14}$$

Thus

$$z = r^*\{1 + [n + 3 + (2/\nu)]\beta t^*\}^{-1/[\nu(n+3)+2]} \tag{15}$$

and Eq. (9) simplifies to

$$\frac{k}{\beta}\left(\frac{d^2f^{n+4}}{dz^2} + \frac{\nu-1}{z}\frac{df^{n+4}}{dz}\right) + \frac{z}{\nu}\frac{df}{dz} + f = 0 \tag{16}$$

$z = 1$ represents the wave front by examining the initial conditions. In order to satisfy the boundary conditions that $f(1) = 0$ at the wave front and $f'(0) = 0$ at the center, we assume

$$f(z) = (1 - z^2)^{\delta} \tag{17}$$

Substituting into Eq. (15) we find that it is the exact solution if

$$\delta = 1/(n + 3) \tag{18}$$

and

$$\beta = 8\nu/3(n + 3) \tag{19}$$

Substituting the values of δ and β into Eqs. (12, 15, and 17) and then into Eq. (8) yields the temperature distribution behind the wave front

$$T = T_0\left[1 + \frac{8}{3}\frac{\nu(n+3)+2}{n+3} \times \frac{E_{R_0}}{E_{M_0}}\frac{l_0c}{r_0^2}t\right]^{-\nu/[(n+3)+2]}(1 - z^2)^{1/(n+3)} \tag{20}$$

with

$$z = \frac{r}{r_0}\left[1 + \frac{8}{3}\frac{\nu(n+3)+2}{n+3}\frac{E_{R_0}}{E_{M_0}}\frac{l_0c}{r_0^2}t\right]^{-1/[\nu(n+3)+2]} \tag{21}$$

From Eq. (21) it follows that the position of the radiation wave front is at

$$r_f = r_0\left[1 + \frac{8}{3}\frac{\nu(n+3)+2}{n+3}\frac{E_{R_0}}{E_{M_0}}\frac{l_0c}{r_0^2}t\right]^{1/[\nu(n+3)+2]} \tag{22}$$

and the wave front propagates into the cold medium at a speed

$$\dot{r}_f = \frac{8}{3(n+3)}\frac{E_{R_0}}{E_{M_0}}\frac{l_0c}{r_0}\left[1 + \frac{8}{3}\frac{\nu(n+3)+2}{n+3} \times \frac{E_{R_0}}{E_{M_0}}\frac{l_0c}{r_0^2}t\right]^{-[\nu(n+3)+1]/[\nu(n+3)+2]} \tag{23}$$

From Eq. (20) the initial temperature distribution for $r \leq r_0$ is found to be

$$T(r,0) = (1 - r^2/r_0^2)^{1/(n+3)} \tag{24}$$

Carrying out the integrals in Eq. (13) we obtain the expressions for total energy enclosed by the wave front

$$Q_0 \begin{cases} r_0CvT_0\left[1 - \frac{1}{3(n+3)} - \frac{n+2}{10(n+3)^2} + \ldots\right] & \text{for } \nu = 1 \\ \pi r_0^2CvT_0(n+3)/(n+4) & \text{for } \nu = 2 \\ \frac{4}{3}\pi r_0^3CvT_0\left[1 - \frac{3}{5(n+3)} - \frac{3(n+2)}{14(n+3)^2} + \ldots\right] & \text{for } \nu = 3 \end{cases} \tag{25}$$

The value of r_0 becomes known once Q_0 and T_0 are specified.

Similarity Solution for $E_R \gg E_M$

Under this condition Eq. (6) becomes

$$\partial T^{*4}/\partial\bar{t} = k[(\partial^2/\partial r^{*2}) + (\nu - 1)/r^*(\partial/\partial r^*)]T^{*n+4} \tag{26}$$

where $\bar{t} = t/r_0^2/l_0c$. The substitutions

$$T^{*4} = \bar{T} \qquad n = 4s + 12 \tag{27}$$

transform Eq. (26) to

$$\partial\bar{T}/\partial\bar{t} = k[(\partial^2/\partial r^{*2}) + (\nu - 1)/r^*(\partial/\partial r^*)]\bar{T}^{s+4} \tag{28}$$

which is similar to Eq. (7). The boundary conditions that the temperature at the wave front and its slope at the center are zero are the same as before. The expressions for Q_0 are obtained by replacing CvT by $\sigma T^4/\rho$ in Eqs. (13), which again vary linearly with $\bar{T}$. Making analogy with the preceding problem we can write out the result for the present case through transformations (27). Thus we obtain the temperature distribution

$$T = T_0\{1 + [2(\nu n + 8)/3n](l_0c/r_0^2)t\}^{-\nu/(\nu n+8)} \times (1 - z'^2)^{1/n} \quad (29)$$

with

$$z' = (r/r_0)\{1 + [2(\nu n + 8)/3n](l_0c/r_0^2)t\}^{-4/(\nu n+8)} \quad (30)$$

The radiation wave front moves in accordance with the law

$$r_f = r_0\{1 + [2(\nu n + 8)/3n](l_0c/r_0^2)t\}^{4/(\nu n+8)} \quad (31)$$

and

$$\dot{r}_f = (8/3n)(l_0c/r_0)\{1 + [2(\nu n + 8)/3n] \times (l_0c/r_0^2)t\}^{-(\nu n+4)/(\nu n+8)} \quad (32)$$

The initial temperature distribution for $r \leq r_0$ and the total energy enclosed are, respectively,

$$T(r,0) = (1 - r^2/r_0^2)^{1/n} \quad (33)$$

and

$$Q_0 = \begin{cases} r_0 \dfrac{\sigma T_0^4}{\rho}\left[1 - \dfrac{4}{3n} + \dfrac{2(4-n)}{5n^2} - \dfrac{2(4-n)(4-2n)}{21n^3} + \ldots\right] & \text{for } \nu = 1 \\ \pi r_0^2 \dfrac{\sigma T_0^4}{\rho}\dfrac{n}{n+4} & \text{for } \nu = 2 \\ \dfrac{4}{3}\pi r_0^3 \dfrac{\sigma T_0^4}{\rho}\left[1 - \dfrac{12}{5n} + \dfrac{6(4-n)}{7n^2} - \dfrac{2(4-n)(4-2n)}{9n^3} + \ldots\right] & \text{for } \nu = 3 \end{cases} \quad (34)$$

Discussion

Self-similar radiation waves are found to propagate into a cold medium at constant density when the material internal energy is assumed to be much greater than the radiation energy and vice versa, corresponding, respectively, to a moderate and a strong nuclear explosion. Equations (20) and (29) show that in either case the temperature has a plateau-like distribution with a sharp front. Corresponding to the temperature gradient, the large pressure gradient leads to the formation of a shock wave. The two equations also indicate that the wave front is sharper in the case of a strong explosion, by virtue of the fact that n is a positive number, which means the opacity decreases with increasing temperature.

Comparing Eqs. (23) and (32) we know that the propagation speed of a radiation wave associated with a strong explosion is much higher than that associated with a moderate one. The rate of decrease in wave speed and the rate of cooling of the gas are much faster in a strong explosion because of its shorter characteristic time. Since the radiation wave is slowing down rapidly, the gasdynamic perturbations caused by the explosion propagating with the speed of sound can catch up to the wave front and our solutions become invalid, as indicated by Andriankin.[3]

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Wall Temperature Effect on Incipient Separation

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For wedge-induced laminar boundary-layer separation on a sharp leading edge flat plate, Hankey[1] has recently shown that for cold walls the constant required in the proportionality proposed by Holden,[2] relating the incipient wedge angle δ_{incip} to the hypersonic interaction parameter $\bar{\chi}$, should be about 1.3, i.e.,

$$M_\infty \delta_{incip} = 1.3\bar{\chi}^{1/2} \quad (1)$$

The extension of the analysis of Ref. 1 has been performed to include the effect of the wall-to-stagnation temperature ratio, T_w/T_0. In the notation of Ref. 1, the temperature ratio enters into the valuation of λ in the equation

$$M_\infty \delta_{incip} = \lambda\bar{\chi}^{1/2} \quad (2)$$

where

$$\lambda = -2K_s[-(\Delta H_{tr}/0.664\Delta K)]^{1/2} \quad (3)$$

For a given temperature ratio the pressure gradient parameter at separation K_s is readily determined from the similar solutions of Ref. 3.

In the evaluation of $\Delta H_{tr}/\Delta K$ the approximation is made, consistent with those already made in the derivation of Eq. (2), that H_{tr} varies linearly between $K = 0$, i.e., the flat plate value, and K_s. Therefore, λ may be calculated for various temperature ratios. The results are given in Fig. 1 together with several values of λ calculated from the experimentally determined δ_{incip} given in the indicated references. Agreement is seen to be good in view of the simplified integral analysis and the difficulties in obtaining accurate values of δ_{incip} from experiments.

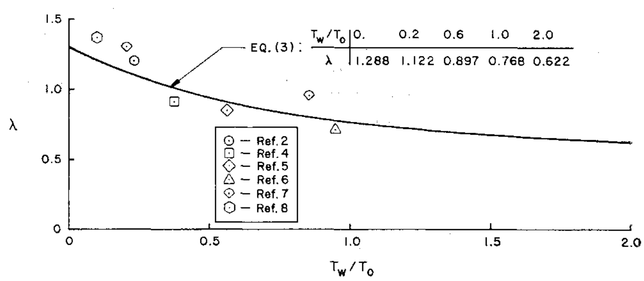

Fig. 1 Dependence of λ in $M_\infty\ \delta_{incip} = \lambda\bar{\chi}^{1/2}$ on T_w/T_0.

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